27 p.573 (Larson)

$$\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$$

<u>Calculator</u> Find the "$\Sigma($" sum function in your TI-89 $\boxed{\text{CATALOG}}$.

This will allow you to find partial sums:

$$S_5 = \sum_{n=1}^{5} \frac{6}{n(n+3)} = \Sigma\left(6/(x^2+3x), x, 1, 5\right) = \frac{235}{84} = 2.798$$

<u>Partial Fractions</u> You need this to actually "find" the infinite sum in 27, 28, 33 - 36.

These infinite series are not geometric, so you can't use $S_\infty = \frac{a_1}{1-r}$.

Start with:

$$n(n+3)\left[\frac{6}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}\right]$$ Partial fraction decomp.

$$6 = A(n+3) + B(n)$$

<u>$n=0$</u> $6 = A(0+3) + \cancel{B(0)}$

$6 = 3A$

$A = 2$

<u>$n=-3$</u> $6 = \cancel{A(0)} + B(-3)$

$6 = -3B$

$B = -2$

So, $$\frac{6}{n(n+3)} = \frac{2}{n} - \frac{2}{n+3}$$

Now, you can actually start to solve the parts.

ⓐ $$\sum_{n=1}^{\infty} \frac{6}{n(n+3)} = \lim_{k\to\infty} \underbrace{\sum_{n=1}^{k}\left[\frac{2}{n} - \frac{2}{n+3}\right]}_{}$$

this is called a <u>telescoping series</u>

$$= \left(\frac{2}{1} - \cancel{\frac{2}{4}}\right) + \left(\frac{2}{2} - \cancel{\frac{2}{5}}\right) + \left(\frac{2}{3} - \cancel{\frac{2}{6}}\right) + \left(\cancel{\frac{2}{4}} - \frac{2}{7}\right) + \left(\cancel{\frac{2}{5}} - \frac{2}{8}\right) + \dots$$

$$+ \left(\cancel{\frac{2}{k}} - \frac{2}{k+3}\right)$$

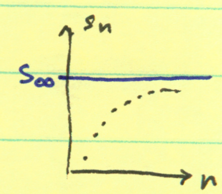
$$= \lim_{k\to\infty}\left[2 + 1 + \frac{2}{3} - \underbrace{\frac{2}{k+3}}_{\to 0}\right]$$

$$= \boxed{3\tfrac{2}{3}} = \boxed{3.667}$$

ⓑ Using the TI-89, the partial sums are:

n	S_n
1	1.5
2	2.1
3	2.433
4	2.648
5	2.798
6	2.909
7	2.994
8	3.063
9	3.118
10	3.164
20	3.394
50	3.551
100	3.608

ⓒ Use a graphing calculator to make a sketch of S_n vs. n



ⓓ As the terms get larger, the partial sums get closer to the infinite sum, but at a slower and slower rate.